

Multipath Diversity

'Rake Receiver'
CDMA.

Frequency Selective:

Channel

ISI
Inter Symbol Interference

Consider the transmission
of symbols $x(0), x(1), \dots, x(n)$.

Output $y(n)$ is,

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \dots + h(L-1)x(n-L+1) + w(n).$$

$$y(n) = \sum_{l=0}^{L-1} h(l)x(n-l) + w(l)$$

Inter Symbol
Interference (ISI)

Frequency Selective
Fading Channel.

For CDMA, $x(n) = \frac{a_0 C_0(n)}{\text{symbol} \times \text{code}}$

$$y(n) = \sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) + w(n)$$

At the receiver, correlating with code $c_0(n)$, we have,

$$\underbrace{\frac{1}{N} \sum_{n=0}^{N-1} y(n) c_0(n)}_{r(0)} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) c_0(n) + \underbrace{\frac{1}{N} \sum_{n=0}^{N-1} w(n) c_0(n)}_{\tilde{w}}$$

$\tilde{w}$ — mean 0
 $\tilde{w}$ — var = σ_w^2/N

$$r(0) = \sum_{l=0}^{L-1} h(l) a_0 \frac{1}{N} \sum_{n=0}^{N-1} C_0(n-l) C_0(n) + \tilde{w}$$

auto correlation of
code $C_0(n)$.

$$= \begin{cases} 1 & \text{if } l = 0 \\ -\frac{1}{N} \approx 0 & \text{if } l \neq 0 \end{cases}$$

$$\Gamma(0) = \underline{h(0) a_0} + \tilde{W}$$

$\uparrow$
 we have ignored terms
 corresponding to $l \neq 0$.
 Since autocorrelation $= \frac{1}{N} \approx 0$

Correlate $y(n)$ with $c_0(n-1)$ i.e.
Delayed version of code.

$$\underbrace{\frac{1}{N} \sum_{n=0}^{N-1} y(n) c_0(n-1)}_{H(1)} = \sum_{l=0}^{L-1} h(l) a_0 \frac{1}{N} \sum_{n=0}^{N-1} c_0(n-l) c_0(n-1) + \underbrace{\frac{1}{N} \sum_{n=0}^{N-1} w(n) c_0(n-1)}_{\tilde{w}(1)}$$

$\tilde{w}(1)$ — mean 0
 $\quad \quad \quad$ var σ_w^2/N

$$\frac{1}{N} \sum_n c_0(n-l) c_0(n-1)$$

$$= \begin{cases} 1 & \text{if } l=1 \\ -\frac{1}{N} \approx 0 & \text{if } l \neq 1 \end{cases}$$

$$r(1) = h(1) a_0 + \tilde{w}(1)$$

because only term
corresponding to $l=1$
survives,

Similarly, one can correlate
with $C_0(n-2), C_0(n-3), \dots, C_0(n-L+1)$

The corresponding outputs can be
expressed as,

$$\begin{bmatrix} r(0) \\ r(1) \\ \vdots \\ r(L-1) \end{bmatrix} = \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix} a_0 + \begin{bmatrix} \tilde{w}(0) \\ \tilde{w}(1) \\ \vdots \\ \tilde{w}(L-1) \end{bmatrix}$$

$$\bar{r} = \bar{h} a_0 + \bar{w}$$

each element
mean 0
var = σ_w^2/N

$$\bar{r} = \bar{h} a_0 + \bar{w}$$

$$\begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix}$$

Coefficients of
Freq selective
channel
= Taps of the
Frequency selective
channel.

Optimal Processing

= Maximal Ratio Combining
(MRC) on $\bar{r}$.

$$= \frac{\bar{h}^H}{\|\bar{h}\|} \cdot \bar{r}$$

"Rake Receiver" in
CDMA

SNR at output of
Rake receiver

$$= \frac{\|h\|^2 P}{\sigma^2/N}$$

Spreading
gain of UDMA.

$$= \frac{N \cdot \|h\|^2 \cdot \frac{P}{\sigma^2}}{N \cdot \|h\|^2 \text{ SNR}}$$

$$= N. \left(|h(0)|^2 + |h(1)|^2 + \dots + |h(L-1)|^2 \right) \cdot \text{SNR}$$

Diversity combining
 ↓
 Multipath Diversity
 in CDMA.

$$\begin{aligned}
 &= N. \| \mathbf{h} \|^2 \text{SNR} \\
 &= \| \mathbf{h} \|^2 \cdot N \text{SNR}
 \end{aligned}$$

BER at high SNR Div order
↓ = L.

$$= 2^{L-1} C_L \cdot \left(\frac{1}{2 \cdot N \cdot \text{SNR}} \right)^L$$

Spreading Gain
of CDMA.

Freq Selective
Taps

When channel taps $h(0) h(1) \dots h(L-1)$
are IID Rayleigh average power = 1.

Example: Consider $L=3$ tap
Frequency selective channel.

$$N = 256, \quad \text{SNR} = 15 \text{ dB.}$$

$$\text{BER} = ?$$

$$10 \log_{10} \text{SNR} = 15 \Rightarrow \text{SNR} = 10^{1.5} \\ = 31.6228$$

$$N = 256, \quad L = 3$$

$$BER = 2^{L-1} C_L \cdot \left(\frac{1}{2 \cdot N \cdot SNR} \right)^L$$

$$= {}^5 C_3 \cdot \left(\frac{1}{2 \times 256 \cdot SNR} \right)^3$$

$$= \frac{10}{8} \cdot \left(\frac{1}{256 \times 10^{1.5}} \right)^3$$

$$= 2.36 \times 10^{-12}$$